13

AN ANALYSIS OF THE J-PHENOMENON IN SCATTERED X-RAYS

Part II

HIRENDRA KUMAR PAL*

DEPARTMENT OF PURE PHYSICS, CALCUTTA UNIVERSITY, CALCUTTA

(*Received November 11, 1964*)

ABSTRACT. Starting with Barkla's idea of 'radiation-atmosphere', where the different constituents of heterogeneous X-rays are considered as an integrated whole and taking into account the various effects due to (i) Compton scattering, (ii) interference of the scattered radiations (predominantly in the forward direction), (iii) absorption of the various constituents of the beam, (iv) narrowness of the primary beam in contrast with the width of the scattered beam, and (v) ionization produced by the radiations received in the ionization chambers, a theoretical analysis has been given to explain, under suitable conditions, most of the experimental results of Barkla and Khastgir (1925-27), both qualitatively and quantitatively, on the comparative study of the primary and the scattered beams of heterogeneous X-rays, (i) when both the beams were filtered through the same increasing thickness of an absorbing material for a given average wavelength of the primary beam and (ii) when both the beams were (a) unintercepted and (b) intercepted by the same thickness of an absorbing material over a wide range of average wavelengths of the primary beam. The laws of the *J*-phenomenon formulated by Barkla have thus been theoretically justified under certain conditions. Barkla's concept of discrete 'levels' of X-ray activity associated with the *J*-phenomenon has not been considered in the present analysis.

INTRODUCTION

In a previous paper the writer (Pal, 1964) published an account of his analysis of the *J*-phenomenon in relation to a single beam of X-rays, irrespective of its genesis. Following Barkla's idea of 'radiation-atmosphere', a heterogeneous beam of X-rays was taken as an integrated whole and the average mass-absorption coefficient of the heterogeneous beam was considered more fundamental than the individual wavelengths. On this basis a consistent and comprehensive interpretation of the main features of the phenomenon, as had been observed by Barkla (1925) was given. The present paper attempts an analytical survey of some other aspects of the same phenomenon, which were revealed by a comparative study of two heterogeneous beams of *X*-rays, chosen out of a primary beam and the same scattered at different angles. In the experimental procedure, this problem was approached from two different directions, as briefly outlined below :

1. Keeping the penetrating power of the incident primary beam constant,

*Present Address. A-91, H. B. Town, P.O. Sodpur, 24 Parganas, W. B.

(a) the scattered beam was observed at a particular angle ϕ between the incident and the scattered beams by intercepting both these beams with an equal thickness x of an absorbing substance (such as Al, Cu, Ag and Sn) and thereafter receiving the transmitted beams into two similar ionization chambers, the ratio of ionizations (S′/P′) in the two chambers for the scattered and the primary beams was found and plotted against thickness x.

(b) the scattered beam at a certain angle ϕ was similarly compared with the beam scattered at an angle ϕ' or 90° and the ratio of ionizations plotted against the intercepting thickness x.

2. The penetrating power of the incident primary beam was progressively increased and for each penetrating power the primary beam was compared with the beam, scattered in any direction ϕ, first, when both the beams were unintercepted and later, when both the beams were simultaneously intercepted by sheets of a definite thickness of some particular absorbing substance. The ratios of ionizations, S/P for the unintercepted and S'/P' for the intercepted beams were determined as in 1. These ratios or their percentage difference was plotted against the corresponding primary mass-absorption coefficient $\left(\frac{\mu}{\rho}\right)$ in aluminium. The results furnished by these experiments may be summarised as follows :

I. The primary and the scattered beams of heterogeneous X-rays or the two beams of heterogeneous X-rays scattered in different directions have either (i) the same absorbability or (ii) a markedly different absorbability, when measured in a particular absorbing substance. This can be expressed as :

$$\left(\frac{\mu}{\rho}\right)_1 - \left(\frac{\mu}{\rho}\right)_2 = 0 \text{ or } a, \text{ or } b, \text{ or } c,$$

where $a, b, c \ldots$ are constants and the subscripts 1 and 2 refer to the primary and the scattered beams or to the scattered beams proceeding in two different directions.

II. A transition from (i) to (ii) in I is possible, under certain conditions, revealing thereby certain discontinuities.

III. When a difference appears in the absorbabilities of the primary and the scattered beams, the percentage difference is independent of the angle of scattering and also of the mass—absorption coefficient of the primary beam.

IV. The position of the discontinuity, when observed appears to depend slightly on the material of the absorbing substance.

These are the laws of the J-phenomenon as had been formulated by Barkla.

The experimental results on the comparative study of progressive filtering of primary and the scattered beams of heterogeneous X-rays further revealed certain anomalous features. These are :

(a) When the scattered beam is different from the primary when measured in certain substances, its absorbability may be and frequently is precisely the same as that of the primary beam when measured in certain other substances.

(b) Even after transmission through substances which show the difference between the primary and the scattered beams, there is still no difference between the two, when measured in certain other substances.

In the present paper, an attempt has been made to give an analysis of the experimental results on the comparative study of the above mentioned two heterogeneous beams and to offer a satisfactory explanation of most of the observed features by taking into account the various factors which are known to produce their respective effects on the ultimate observations. We shall refer to the experi-mental results which had been published by C. G. Barkla and S. R. Khastgir 1925b), 26b, 27)

BACKGROUND OF THE ANALYSIS

For a theoretical analysis of the results of experiments in question we have to consider four different types of problems, viz. (i) scattering, (ii) interference, (iii) absorption and (iv) ionization. Each of these effects is known to have a direct bearing on the wavelength of the radiation concerned.

When the X-ray beam experimented upon is heterogeneous, as in the present case, a theoretical approach is beset with many difficulties. For example, a spectrum with a vast multitude of wavelengths has to be confronted. To make things worse, the energy-distribution function over the constituents is not known with certainty and is likely to vary with the tube generating the X-rays. Further, a part of the spectrum, towards the longer wavelength-side, is absorbed while traversing its various air-paths, even before reaching the measuring instruments. This sets an uncertain limit to the longer wavelengths in the spectrum under study and hence any summation-operation carried between limits, one of which is ill-defined, cannot be expected to yield either comparable or reliable results.

It is, therefore, at once obvious that such a heterogeneous complex beam of X-rays should, in a mathematical analysis, be best treated as an integrated whole without any special reference to the individual monochromatic constituents and this should have an 'average' property which, to some extent, is measurable by what is known as the 'mass-absorption coefficient'. This had been frequently stressed by Barkla. On the basis of this, it becomes possible to assign one wave-length—an average one—to the whole beam, as though it were homogeneous.

(i) *Scattering*

According to the classical theory, the intensity scattered by a single 'free' electron at an angle ϕ with the primary beam, is given by the wellknown equation

$$I_s = K_1 I_P (1 + \cos^2 \phi) \qquad \dots \quad (1)$$

where $K_1 = e^4 / / 2r^2 m^2 c^4 =$ constant

and $I_P =$ Intensity of the primary beam.

For the whole atom, the expression is to be multiplied by Z^2 or Z, (where Z is the atomic number) according as the packing of the electrons inside the atom is very close or otherwise.

On the quantum theory, the Compton-change of wavelength on scattering, is given by—

$$\partial \lambda_\varphi = \frac{h}{mc} \text{ vers } \phi = \delta\lambda_{90}{}^\circ \text{ vers } \phi \qquad \dots \quad (2)$$

and the scattering function, as given by Breit, Dirac (1926) and others can be written as :

$$I_\varphi = I_s \left(1 + \frac{h\nu}{mc^2} \text{ vers } \phi\right)^{-3} = I_s \left(1 + \frac{\delta\lambda_\varphi}{\lambda}\right)^{-3} \qquad \dots \quad (3)$$

(The notations used in the above equations have their usual significance.)

Our method of analysis is as follows : The heterogeneous primary beam is regarded as a whole and defined by an average wavelength. The scattered radiation consists of two parts, one of which is 'modified' and the other 'unmodified' after scattering. The modified portion is assumed to follow the scattering function expressed in (3) and the unmodified portion, naturally that given in (1). The effects of interference and absorption on each of the two scattered intensities are then considered separately and finally the ionizations produced by the corresponding transmitted parts. These ionizations when added up give the resultant ionization occuring inside the ionization chamber for the scattered beam. In the case of the ionization in the primary chamber only the effect of absorption on the primary intensity as a whole is considered.

In connection with the scattering of a heterogeneous beam of X-rays the Compton-change of wavelength which should be appropriately ascribed to the modified portion of the scattered rays is, of course, that given by (2). Such a change in a particular direction is always independent of the incident wavelength. Hence an average wavelength λ, when modified, should have an average wavelength $\lambda + \delta\lambda_\rho$.

As for the percentage modification (which is defined as : 100 $\times$ fraction of the incident intensity modified) occuring in a scattering process, it is learnt from the experiments of Backhurst (1934) with crystal-selected monochromatic X-rays, that for very hard rays ($\lambda = 0.25$ Å) and light scattering atoms such as beryllium or carbon, the modification, at $\phi = 150°$, is approximately 100%. But this figure steadily diminishes either as the wavelength or as the atomic number of the scattering element is increased. The data for $\phi \gtreqless 90°$, however, are lacking.* In the case of a heterogeneous complex beam, we are, of course, concerned with an average percentage modification corresponding to an average wavelength.

Let C_φ represent the fraction of the incident intensity which is modified on being scattered at an angle ϕ, so that $(1-C_\varphi)$ represents the corresponding fraction which is unmodified. Therefore, from** (1) and (3), the modified scattered intensity in the direction ϕ is given by

$$I'_\varphi = KI_pC_\varphi\,(1+\cos^2\phi)\left(1+\frac{\delta\lambda_\varphi}{\lambda}\right)^{-3} \quad \ldots \quad (4)$$

and the unmodified scattered intensity in the same direction

$$I''_\varphi = KI_p(1-C_\varphi)(1+\cos^2\phi) \quad \ldots \quad (5)$$

where K = constant.

(ii) *Interference*

The effect of interference of the scattered radiations from the scattering electrons has been observed to play an important role in determining the nature of results in the absorption experiments with heterogeneous scattered X-rays. Indeed, in course of a previous publication (Pal, 1949), the writer definitely showed that the 'excess scattering' in the forward direction, arising from interference of the scattered radiations, could introduce anomalies into the results even to the extent of concealing or refuting the Compton-effect. So far as the J-phenomenon is concerned, the effect of interference does not seem to have received due consideration from the workers on the subject. Excess scattering, owing to the preponderance of longer wavelengths in the heterogeneous scattered beam, produces a general softening of this beam increasing its average wavelength. This has to be taken into account, remembering that the effect increases with an increase in the incident wavelength or in the atomic number of the scattering element and with a decrease in the scattering angle ϕ (Pal, 1948).

The effect due to interference is two-fold : There is (i) an enhancement of the scattered intensity and (ii) an increase in the average wavelength of the scattered beam.

---

* Although Backhurst's results lack in data for $\varphi \gtreqless 90°$, yet it will not be unreasonable to expect that they are in line with those for $\varphi = 150°$, in a general way at least.

** Here we neglect the small amount of polarization that may be present in the incident radiation.

Let us represent the enhancement of intensity by a factor $\{1+\psi(\phi, \lambda)\}$ for a given scatterer. Let us also put:

$$f(\phi, \lambda) = (1+\cos^2\phi)\{1+\psi(\phi, \lambda)\} \qquad \dots \quad (6)$$

Now, when $\phi \gtreqless 90°$, the effect of interference is negligible, so that $\psi = 0$ and we get

$$f = 1+\cos^2\phi = 1, \text{ when } \phi = 90°.$$

We also get

$$f' = \frac{df}{d\lambda} = 0 \qquad \dots \quad (7)$$

Let the increase in the average wavelength due to interference be represented by Δ_φ and that produced by the Compton-effect by $\delta\lambda_\varphi$, so that the total increase in the average wavelength for the modified scattered rays is given by

$$\Delta_\varphi = \Delta_\varphi + \delta\lambda_\varphi \qquad \dots \quad (8)$$

and that for the unmodified scattered rays by Δ_φ only. Here it is recognised that, strictly speaking, the contributions by interference are different for the modified and unmodified rays. But the difference is regarded as one of second-order smallness and hence neglected.

After interference has taken place, the intensities of the scattered radiation, in the direction ϕ, are obtained from (4), (5) and (6), Thus for the modified part, we get

$$I'_\varphi = KI_P C_\varphi (1+\cos^2\phi)\left(1+\frac{\delta\lambda_\varphi}{\lambda}\right)^{-3}\{1+\psi(\phi, \lambda+\delta\lambda_\varphi)\}$$

$$= KI_P C_\varphi \left(1+\frac{\delta\lambda_\varphi}{\lambda}\right) f(\phi, \lambda+\delta\lambda_\varphi \qquad (9)$$

and for the unmodified part,

$$I''_\varphi = KI_P(1-C_\varphi)(1+\cos^2\phi)\{1+\psi(\phi, \lambda)\}$$

$$= KI_P(1-C_\varphi) f(\phi, \lambda) \qquad \dots \quad (10)$$

(iii) *Absorption*

When a heterogeneous beam of X-rays traverses a thickness x of an absorbing sheet of matter it is hardened up owing to its softer constituents being more readily absorbed. As x increases, (μ/ρ) for the emergent beam decreases, so also its average wavelength λ. The rate of variation of (μ/ρ) or of λ, however, becomes smaller and smaller with an increase in x, till at a certain stage, this variation is practically indistinguishable for any further increase in x, so that the course of the $(\mu/\rho, x)$—curve is then practically parallel to the x—axis, signifying that the transmitted rays have become more or less homogeneous.

Critical absorbing thickness : If two heterogeneous beams with slightly different penetrating powers (such as a primary and the corresponding scattered beam) be each passed through a thickness x of an absorbing material, their (μ/ρ)-values, on emergence, should generally exhibit a difference. But as the softer of the two beams is absorbed in a somewhat greater proportion, the rate of fall of (μ/ρ) with increasing x, for the softer beam, will be steeper than that for the beam which is less soft. This means that the difference $\delta(\mu/\rho)$ between them gradually diminishes as x increases, till for a thickness equal to or greater than a certain value x_c, this difference is practically indistinguishable. The minimum absorbing thickness x which, so to say, brings down both the beams to the same level of absorbability, will be hereafter, called *critical absorbing thickness*

Thus an important property of critical thickness is that,

if $x \gtreqless x_c$

$$\left.\begin{aligned} \left[\delta\left(\frac{\mu}{\rho}\right)\right]_x &= 0 \\ \text{or} \qquad (\delta\lambda)_x &= 0 \end{aligned}\right\} \qquad \ldots (11)$$

where the subscript x signifies to transmission of each of the beams concerned through an absorbing substance of thickness x.

It is easy to see that the critical thickness for an absorber should increase with the hardness of the radiation and for any radiation should decrease with an increase in the atomic number of the absorbing element.

Mass-absorption coefficient : This coefficient, in the case of a monochromatic beam of X-rays, follows a linear relation with λ^3. Thus

$$\frac{\mu}{\rho} = A + B\lambda^3 \qquad (12)$$

where A and B are constants depending upon the material of the absorber.

The first term A arises from the energy absorbed by recoil electrons and is called the 'scattering absorption coefficient' and the second term B is due to the energy absorbed by the ejected photo-electrons and is called the 'true absorption coefficient'.

For a *narrow* beam of X-rays, like the primary beam, the energy absorbed by recoil-electrons is practically lost to the beam, whereas for a *wide* beam, a part of the scattered energy will remain lodged inside the beam and add to its intensity. Thus the (μ/ρ) for a narrow beam is expected to exceed that for a wide beam. The scattered beam in the experiments under review was always very wide. For such a wide beam A in (12) would be reduced to a somewhat smaller value A'. The amount of reduction would depend on the geometry of the measuring instrument. This had actually been varified experimentally by Bachem (1923).

The extrapolated values of A and B for a few chemically pure* elements are given in the following Table I. In the 5th column of this table, are also supplied the corresponding maximum values of absorbing thickness employed in the experiments under review, for quick reference.

TABLE I

Absorbing element	Range of wavelength in A.U.	A	B	x(max.) in cm.
Al	0.2 —0.7	.13	· 14.1	0.150
Cu	0.2 —0.4	.25	153	0.009
	0.4 —0.7	1.5	140	
Ag	0.3 —0.4	3.5	551	0.012
	0.5 —0.6	.5	84.6	
Sn	0.25—0.3	2.5	600	0.010
	0.3 —0.7	?	?	
Au	0.25—0.55	?	?	0.008

Transmitted Intensity : In the case of a narrow monochromatic beam of X-rays, the transmitted intensity is givem by

$$I_x = I_0 \exp\left(-\frac{\mu}{\rho}\, x\right) = I_0 \exp\{-(A+B\lambda^3)x\} \qquad \dots \quad (13)$$

where I_0 is the intensity of the incident beam. For a wide beam A is replaced by A'.

To calculate the transmitted intensity in the case of a heterogeneous beam, we have to substitute for λ in (13) the average wavelength 'L' inside the absorbing sheet, which is the mean of the average incident wavelength λ and the average transmitted wavelength λ_x. Thus the required average is given by

$$\therefore \qquad L = \tfrac{1}{2}(\lambda+\lambda_x), \text{ (since } x \text{ is small)}$$

We shall here put : $\lambda_x = \lambda(1-2\omega)$ where $\omega = \omega(x, \lambda, Z)$. ... (14)

$$\therefore \quad L = \tfrac{1}{2}\{\lambda+\lambda(1-2\omega)\}$$
$$= \lambda(1-\omega) \qquad \dots \quad (15)$$

Substituting this in (13),

$$I_x = I_0 \exp\{-(A+BL^3)x\}$$
$$= I_0[\exp(-Ax)] \exp\{-Bx\lambda^3(1-\omega)^3\} \qquad \dots \quad (16)$$

* Traces of impurities as are usually present in the commercial varieties of the metals are apt to affect the values of A and B considerably.

'Disparity' :

For λ we have put :

$$\lambda_x = \lambda(1-2\omega)$$

$\therefore$ for $\lambda+\delta\lambda$ we write

$$(\lambda+\delta\lambda)_x = (\lambda+\delta\lambda)(1-2\omega')$$

so that when $x = 0$, $\omega = 0$ and $\omega' = 0$

and when $x > x_c$, $\omega \approx$ const. and $\omega' \approx$ const.

For small values of x, ω and ω' are supposed to be small compared to unity.

Now, $(\lambda+\delta\lambda)_x = (\lambda+\delta\lambda)\cdot(1-2\,\omega')$

$$= \lambda\left\{1+\frac{\delta\lambda}{\lambda}-2\omega'\right\}, \text{ (neglecting second and higher order smallness)}$$

and
$$\lambda_x = \lambda(1-2\omega)$$

$$\therefore \quad (\lambda+\delta\lambda)_x/\lambda_x = 1+\frac{\delta\lambda}{\lambda}-2(\omega'-\omega), \qquad \dots \quad (17)$$

(neglecting second and higher order smallness)

Thus
$$\frac{(\lambda+\delta\lambda)_x-\lambda_x}{\lambda_x} = \frac{\delta\lambda}{\lambda}-2(\omega'-\omega) = q_x \text{ (say)} \qquad \dots \quad (18)$$

Here q_x is called the 'disparity"-term which is defined as the fractional difference between the average transmitted wavelengths of the two beams of slightly different average wavelengths after passing through a certain thickness x of an absorbing substance.

Also from (18) we can write

$$\frac{\delta\lambda}{\lambda}-(\omega'-\omega) = \frac{1}{2}\left(q_x+\frac{\delta\lambda}{\lambda}\right) \qquad \dots \quad (19)$$

Properties of q_x:

(a). When $x = 0$, $\omega' = 0 = \omega$ $\therefore$ $q_0 = \frac{\delta\lambda}{\lambda}$

(b). When $x \gtreqless x_c$, $(\delta\lambda)_x = 0$ (*vide eqn.* 11), so that

$$(\lambda+\delta\lambda)_x/\lambda_x = 1 \quad \text{and} \quad q_x = 0$$

Also $2(\omega'-\omega) = \delta\lambda/\lambda$ from (18).

(c) When the two beams compared are equally hard

$$\delta\lambda = 0, \quad \omega = \omega' \quad \therefore \text{ from (18), } q_x = 0$$

(d) For homogeneous beams, $\omega' = 0 = \omega$

$\therefore$ from (18), $q_x = \frac{\delta\lambda}{\lambda} =$ constant

(e) It will be sometimes necessary to know how the quantity, $(q_x \cdot x)$, varies with x. For this, we note that this quantity is always positive, having a value 0 at $x = 0$ and at $x \gtreqless x_c$. Therefore, somewhere between $x = 0$ and $x = x_c$, $(q_x \cdot x)$ has a maximum value :

(f) As λ decreases, the quantity $2(\omega' - \omega)$ i.e. $\left(\frac{\delta\lambda}{\lambda} - q_x\right)$ increases. For, when λ is large, $\omega' \gtrsim \omega$ and when λ is small, $\omega' > \omega$.

(iv) *Ionization in SO_2 inside the ionization chamber*

This has been found to be proportional to the intensity and the cube of the wavelength of the rays concerned. Thus if I_x represent the ionization produced by the radiation transmitted through a thickness x of the absorbing material, then

$$
\begin{aligned}
\mathrm{I}_x &= g I_x \lambda_x^3 \\
&= g I_x \lambda^3 (1-2\omega)^3 \qquad \ldots \quad (20)
\end{aligned}
$$

where g = Constant.

SECTION I.

COMPARISON OF THE SCATTERED AND THE PRIMARY RADIATIONS AT CONSTANT INCIDENT WAVELENGTH AFTER PROGRESSIVE FILTERING

A. *Evaluation of the ratio (S'/P')* :

Let S' represent the ionization produced by the scattered beam in the direction ϕ and P' that by the primary beam, after they have traversed, each a thickness x of the absorbing material.

Then

$$S' = S'_1 + S'_2$$

where the subscript 1 denotes the contribution by the modified portion of the rays and the subscript 2 that by the unmodified, so that

$$(S'/P')\varphi = (S'_1/P')\varphi + (S'_2/P')\varphi$$

Now, neglecting the absorption inside the scatterer (supposed to be light and thin), and making use of equations (8), (9), (10), (16), and (20) we may write

$$P' = I_P[\exp(-Ax)][\exp\{-B\lambda^3(1-\omega)^3 x\}]g_0\lambda^3(1-2\omega)^3$$

$$S'_1 = KI_P C\varphi \left(1+\frac{\delta\lambda\varphi}{\lambda}\right)^{-3} f(\phi, \lambda+\delta\lambda\varphi)[\exp(-A'x)].$$

$$[\exp\{-B(\lambda+\Delta\varphi)^3(1-\omega'')^3 x\}]g(\lambda+\Delta\varphi)^3(1-2\omega'')^3$$

and

$$S'_2 = KI_P(1-C_\varphi)f(\phi, \lambda)[\exp(-A'x)].$$

$$[\exp\{-B(\lambda+\Delta\varphi)^3(1-\omega')^3 x\}]g(\lambda+\Delta\varphi)^3(1-2\omega')^3$$

where g, a constant in (20) corresponds to the scattered beam and g_0 to the primary beam and ω, ω', ω'' refer to the primary beam, the unmodified scattered beam and the modified scattered beam respectively.

Thus $(S'_1/P')\varphi = K(g/g_0)\, C\varphi \left(1+\dfrac{\delta\lambda\varphi}{\lambda}\right)^{-3} f(\phi, \lambda+\delta\lambda\varphi)[\exp\{(A-A')x\}].$

$$[\exp - Bx\{(\lambda+\underline{\Delta}\varphi)^3(1-\omega'')^3-\lambda^3(1-\omega)^3\}](\lambda+\underline{\Delta}_\varphi)^3(1-2\omega'')^3 \Big/ \lambda^3(1-2\omega')^3$$

or $$\left(\frac{S'_1}{P'}\right)_\varphi = K'C\varphi[\exp\{(A-A')x\}]f(\phi, \lambda)\left\{1+\frac{f'\delta\lambda\varphi}{f}\right\}.$$

$$\left[\exp -3Bx\,\lambda^3\left\{\frac{\Delta\varphi}{\lambda}-(\omega''-\omega)\right\}\right]\left(1-3\frac{\delta\lambda\varphi}{\lambda}\right)\left\{1+\frac{3\Delta_\varphi}{\lambda}-6(\omega''-\omega)\right\},$$

putting $K' = K{\cdot}g/g_0 =$ Const. and neglecting second and higher order smallness. This can be approximately written as :

$$(S'_1/P')_\varphi = K'C_\varphi\,[\exp\{(A-A')x\}]\,f(\phi, \lambda)\left[1+\frac{f'}{f}\,\delta\lambda_\varphi -3Bx\lambda^3\left\{\frac{\Delta_\varphi}{\lambda}-(\omega''-\omega)\right\}\right.$$
$$\left. +3\left\{\frac{\Delta_\varphi}{\lambda}-2(\omega''-\omega)\right\}\right]$$

Proceeding similarly we may evaluate

$$(S'_2/P')_\varphi = K'(1-C_\varphi)[\exp\{(A-A')x\}]f(\phi, \lambda)\left[1-3Bx\lambda^3\left\{\frac{\Delta_\varphi}{\lambda}-(\omega'-\omega)\right\}\right.$$
$$\left. +3\left\{\frac{\Delta_\varphi}{\lambda}-2(\omega'-\omega)\right\}\right]$$

Now considering the Compton-change of wavelength $\delta\lambda_\varphi$ and the wavelength change Δ_φ due to interference we can write :

$$q_x = \frac{\Delta_\varphi}{\lambda}-2(\omega''-\omega), \text{ where } \underline{\Delta}_\varphi = \Delta_\rho+\delta\lambda_\varphi \text{ (\textit{vide} eqn. 8)}$$

or $$\frac{\Delta_\varphi}{\lambda}-(\omega''-\omega) = \tfrac{1}{2}\left(q_x+\frac{\Delta_\varphi}{\lambda}\right)$$

Now putting $$p_x = \frac{\Delta_\varphi}{\lambda}-2(\omega'-\omega)$$

we get $$\frac{\Delta_\varphi}{\lambda}-(\omega'-\omega) = \tfrac{1}{2}\left(p_x+\frac{\Delta_\varphi}{\lambda}\right)$$

Substituting these values in the above expressions we have,

$$(S'_1/P')_\varphi = K'C_\varphi\,[\exp\{(A-A')x\}]f(\phi,\lambda)\cdot$$
$$\left[1+\frac{f'}{f}\delta\lambda_\varphi-\frac{3}{2}Bx\lambda^3\left(q_x+\frac{\Delta_\varphi+\delta\lambda_\varphi}{\lambda}\right)+3\left(q_x-\frac{\delta\lambda_\varphi}{\lambda}\right)\right]$$

and

$$(S'_2/P')_\varphi = K'(1-C_\varphi)[\exp\{(A-A')x\}]f(\phi,\lambda)\cdot$$
$$\left[1-\frac{3}{2}Bx\lambda^3\left(p_x+\frac{\Delta_\varphi}{\lambda}\right)+3p_x\right]$$

Adding up the above two expressions we have finally

$$(S'/P')_\varphi = (S'_1/P')_\varphi+(S'_2/P')_\varphi$$
$$= K'f(\phi,\lambda)[\exp\{(A-A')x\}]\left[1+C_\varphi\frac{f'}{f}\,\delta\lambda_\varphi\right.$$
$$-\frac{3}{2}Bx\lambda^3\left\{\frac{\Delta_\varphi}{\lambda}+p_x+C_\varphi\left((q_x-p_x+\frac{\delta\lambda_\varphi}{\lambda}\right)\right\}$$
$$\left.+3\left\{p_x+C_\varphi\left((q_x-p_x-\frac{\delta\lambda_\varphi}{\lambda}\right)\right\}\right] \quad \ldots \quad (21)$$

B. *Progressive Absorption*, $\phi < 90°$

From the expression (21) it is noted that the curve $(S'/P')_\varphi$ plotted against small values of x, should steadily slope down. For, p_x and q_x-p_x (which is positive) diminish, while $p_x.x$ and $(q_x-p_x).x$ increase with increasing x, and the exponential term is nearly equal to unity, at least for aluminium and copper. This result is in agreement with those illustrated for $\phi = 30°$, $60°$ and $90°$ in Fig 4, p. 652 Barkla and Khastgir (1926b)

C. *Progressive Absorption*, $\phi = 90°$.

When $\phi = 90°$, we have $f = 1$, $f' = 0$, $\Delta_\varphi = 0$. Taking $\omega' = \omega$, we get $p_x = 0$. Putting now $q_x = r_x$, we write $r_x = \frac{\delta\lambda 90°}{\lambda} - 2(\omega''-\omega)$.

Therefore, substituting these in (21) we have

$$(S'/P')_{90°} = K'\left[1-3C_{90°}\left\{\frac{B}{2}x\lambda^3\left(r_x+\frac{\delta\lambda 90°}{\lambda}\right)+\left(\frac{\delta\lambda 90°}{\lambda}-r_x\right)\right\}\right]\exp\{(A-A')x\}$$

$$= K'\left[1-3C_{90°}\left\{\frac{B}{2}x\lambda^3\left(r_x+\frac{\delta\lambda 90°}{\lambda}\right)+\left(\frac{\delta\lambda 90°}{\lambda}-r_x\right)\right\}+(A-A')x\right] \quad \ldots \quad (22)$$

since $(A-A')x$ is small as compared to unity.

---

* Usually the terms with coefficient $Bx\,\lambda^3$ are very small compared to unity and can be neglected in equation (21). This will greatly simplify all subsequent calculations leading, however, to identical results.

Let us now consider different absorbers. The following cases are considered : (i) For Al and Cu absorbers the term $(A-A').x$ is negligibly small, so that the course of the curve $(S'/P')_{90^\circ}$ plotted against small values of x, should be descending towards the right, as for $\phi < 90^\circ$.

(ii) For Ag and Sn, on the contrary, the term $(A-A')\cdot x$ may not be negligible (*Vide* Table I), so that it will counteract the preceding term in equation (22) and may actually balance the same within limits of experimental error, for a suitable wavelength; in which case the only outstanding term on the right hand side of the equation is $K' =$ Const. This means that the $(S'/P')_{90^\circ}-x$ graph will be a horizontal straight line, as though the scattered radiation were 'unmodified' (*Vide* Eqn 31). These results are illustrated in Fig. 2., p. 646, Barkla and Khastgir (1926b).

Similar and confirmatory graphs for the Ag-absorber, with a small downward trend at the right hand side extremity, were obtained by the present writer also, with radiations, $\lambda = 0.55$ Å and 0.60 Å. With harder radiations, $\lambda = 0.46$ Å (unfiltered) and 0.38 Å (filtered), neutralisation as above, was only partial and the curve sloped down to the tune of 10 and 15% respectively for $x = 0.009$ cm exactly as theory would have it.

As $(A-A') \approx 0$ for Cu, we can hardly expect a horizontal straight line for the curve $(S'/P')_{90^\circ}$ against x, except in the region of soft unmodified rays ($C_{90^\circ}=0$). Al may give such a horizontal line graph but that also with radiations which are not modified for some reason or other.

To explain the dual behaviour simultaneously, on the part part of Sn, in the reference mentioned above, the inference that commends itself is that C_{90° had changed in the process of scattering. It might also be that the incident radiation concerned had passed through a state of instability during that particular experiment, causing a fluctuation of the hardness of the rays and therewith of C_{90°. Such a possibility, of course, cannot be ruled out, especially for gas tubes working continuously for a long time. A fluctuation of C_{90° may also be held responsible for the so-called 'sub-level' in Fig. 4 of the same reference.

The foregoing evidences led Barkla and Khastgir to hold that the same radiation could be a modified radiation when tested with Al or Cu—absorber, but an unmodified one when tested with Ag and also, sometimes modified and sometimes unmodified when tested with Sn. In the light of the present analysis we are, however, inclined to the view that scattering was undoubtedly accompanied by a modification of the Compton-type in every case treated above and that the observed difference was due merely to two mutually opposing tendencies at work later during absorption. These tendencies practically balanced each other, as a matter of chance, in the case of Ag and Sn; whereas, for Al and Cu absorbers, such a balance was impossible, the effect then being practically unilateral. Although a truly unmodified' radiation should mean an almost horizontal straight line in many cases (*Vide* eqn. 31), yet the converse is not necessarily always true. In other words, a Compton-change of wavelength which has actually taken place at scattering, may not be invariably evident from a concomitant change in the absorbability of a heterogeneous scattered beam tested as above.

The experiments with compound absorbers yielded anomalous results which require explanation [*Vide* Fig. 3, *p* 649, Barkla and Khastgir (1926b). If a second absorber of thickness 'y' and made of a different material, be placed behind the first, i.e. on the emergent sides of the primary and secondary beams, corresponding additional terms have to be introduced into the expression (22) to account for the extra absorption. Denoting the two successive absorptions by double dashes, we have at $\phi = 90°$.

$$(S''/P'')_{x+y} = K'\Big[1-3C_{90°}\Big\{\frac{B_1}{2}x\lambda^3\Big(r_x+\frac{\delta\lambda 90°}{\lambda}\Big)+\Big(\frac{\delta\lambda 90°}{\lambda}-r_x\Big)\Big\}+(A_1-A_1')x$$

$$-3C'_{90°}\Big\{\frac{B_2}{2}y\lambda^3_x\Big(\bar{r}_y+\frac{(\delta\lambda 90°)_x}{\lambda_x}\Big)+\Big(\frac{(\delta\lambda 90°)_x}{\lambda_x}-\bar{r}_y\Big)(+(A_2-A'_2)y\Big\}\Big] \quad \ldots \quad (23)$$

where subscripts 1 and 2 refer to the first and second absorbers respectively and $\bar{r}_y$ corresponds to the beams which pass through the second absorber after transmission through the first.

Cor. 1. If $x < x_c$

$$(S''/P'')_{x+y} < (S'/P')_x,$$

since the quantity within each curled bracket is essentially positive and further $(A_1-A'_1)\cdot x$ and $(A_2-A'_2)\cdot y\cdot$ are small. Hence $(S''/P''-x)$ graph lies below the $(S'/P'-x)$ graph, both sloping downwards. This is illustrated for paper and paper *plus* aluminium in fig. 3 C of the above reference where $y_{Al} = 0.048$ cm.

Cor. 2. If y is very small, i.e. $y \approx 0$.

then $(S''/P'')_{x+y} \approx (S'/P'\cdot)_x$ since $\bar{r}_y \approx \bar{r}_0 \approx \dfrac{(\delta\lambda_{90})_x}{\lambda_x}$.

This means that the two ratios plotted against x, give coincident graphs as in Fig. 3A or 3B of the above reference, where x represents the running thickness of Al and y the constant thickness (0.002 cm) of Ag or of Sn. In an analogous manner, paper and paper *plus* Ag.(0.002 cm.) also give concurrent graphs as in Fig. 3C.

Thus if 0.002 cm. thickness of Ag or of Sn is regarded as very small for the purpose of absorption, the principal feature of Fig. 3. (A, B and C) can be explained easily.

Cor. 3. If $x \gtreqless x_c$ and y is small but not very small then also $(S''/P'')_{x+y} = (S'/P')_x$ since $r_x = 0$, $(\delta\lambda 90°)_x = 0$ and $\bar{r}_y = 0$. $(A_1 - A'_1)\cdot x$ and $(A_2 - A'_2)\cdot y$ are supposed to be very small also.

SECTION II

COMPARISON OF THE SCATTERED RADIATIONS AT CONSTANT PRIMARY WAVELENGTH: $\phi = \phi°$ AND $\phi = 90°$

A. *Evaluation of the ratio* $S'_\varphi / S'90° = R'_\varphi$ *(say).*

Let S'_φ represent the ionization produced by the scattered radiation in the direction ϕ, after it has passed through a thickness 'x' of an absorbing substance and $S'90°$ the corresponding quantity, in the direction $\phi = 90°$. Then $R'_\varphi = S'_\varphi / S'90° = (S'/P')_\varphi / (S'/P')90°$. Substituting the values of the ratios from equations (21) and (22) we have, after neglecting second and higher order smallness

$$R'_\varphi = f(\phi, \lambda)\Big[1 + C_\varphi \frac{f'}{f}\delta\lambda_\varphi$$

$$- \frac{3}{2} Bx\lambda^3\Big\{\frac{\Delta_\varphi}{\lambda} + p_x + C_\varphi\Big(q_x - p_x + \frac{\delta\lambda_\varphi}{\lambda}\Big) - C90°\Big(r_x + \frac{\delta\lambda 90°}{\lambda}\Big)\Big\}$$

$$+ 3\Big\{p_x + C_\varphi\Big(q_x - p_x - \frac{\delta\lambda_\varphi}{\lambda}\Big) + C90°\Big(\frac{\delta\lambda 90°}{\lambda} - r_x\Big)\Big\}\Big] \qquad \ldots \quad (24)$$

If R_φ represent the ratio for the unintercepted scattered beams its value can be deduced by putting $x = 0$ in the above eqn.(24).

Thus

$$R_\varphi = f(\phi, \lambda)\Big[1 + C\phi \frac{f'}{f}\delta\lambda_\varphi + 3\Big\{p_0 + C_\varphi\Big(q_0 - p_0 - \frac{\delta\lambda_\varphi}{\lambda}\Big) + C90°\Big(\frac{\delta\lambda 90°}{\lambda} - r_0\Big)\Big\}\Big]$$

Since $\quad r_0 = \dfrac{\delta\lambda 90°}{\lambda}$ and $\underline{\Delta}_\varphi = \Delta_\varphi + \delta\lambda_\xi$ (*Vide* eqn. 8).

$$R_\varphi = f(\phi, \lambda)\Big[1 + C_\varphi \frac{f'}{f}\delta\lambda_\varphi + 3\frac{\Delta_{\varphi}}{\lambda}\Big] \qquad \ldots \quad (25)$$

This gives the ratio of ionizations produced by the unintercepted scattered beams one of which is in the forward direction, $\phi < 90°$. In the backward direction, $\phi > 90°$, we have

$$f(\phi, \lambda) = 1 + \cos^2\phi$$

$$f' = 0 \text{ and } \Delta_\varphi = 0 \text{ (Vide eqn. 7), so that}$$

$R_\varphi = 1 + \cos^2\phi$. This is same for all wavelengths.

This affords an easy experimental test of the accuracy of the foregoing analysis and was verified by the writer at $\phi = 150°$, for which R_ϕ should be equal to 1.75. The experimental data (Pal, 1948 and 1950) in this connection, with light paraffin wax scatterer (for which the treatment is more aptly valid), are presented in the following Table II. In it, the observed ratio of the ionizations in the directions concerned, the ratio corrected for polarization of the incident beam and the ratio further corrected for the obliquity of the scattered pencil are entered in the 4th, 5th, and 6th. columns respectively. The last column gives the divergence of each ratio from the mean of the ratios.

TABLE II

Equiv. λ in A.U.	P/U*	(2P/U) $\cos^2\phi$	Uncorrected ratio	Ratio corrected for polarization	Ratio next corrected for obliquity	Divergence% of the ratio from the mean
0.77	0.100	0.150	1.79	1.63	1.65	−2.3
0.49	0.023	0.036	1.745	1.71	1.73	+2.3
0.44	0.016	0.024	1.72	1.69	1.71	+1.2
0.34	0.017	0.026	1.725	1.69	1.71	+1.2
0.275	0.026	0.039	1.705	1.66	1.68	−0.6
0.25	0.032	0.048	1.69	1.63	1.65	−2.3
0.225	0.045	0.068	1.65	1.57	1.59	
0.33	0.042	0.063	1.72	1.65	1.67	−1.2
..	..	..	..	..	Mean 1.69	

* P = Planepolarized intensity. U = unpolarized intensity.

It will be noted from the above table that the corrected ratio is fairly constant, lying within $\pm 2.3\%$ of the mean value (1.69). The small defect of the mean ratio from the theoretical 1.75, may be attributed to the absorption (neglected) occuring inside the scatterer as also inside the thin aluminium sheet (thickness .01 cm.) covering the window of the ionization chamber. For full details of the corrections reference may be made to the previous papers (Pal, 1948 and 1949) published by the author.

B. *Progressive Absorption*

(a) In the backward direction, $\phi > 90°$, there is no interference and the equation (24) may be written down as :

$$R'_\varphi = (1+\cos^2\phi)\left[1-\frac{3B}{2}x\lambda^3\left\{C_\varphi\left(q_x+\frac{\delta\lambda_\varphi}{\lambda}\right)-C_{90°}\left(r_x+\frac{\delta\lambda 90°}{\lambda}\right)\right\}\right.$$
$$\left.+3\left\{C_\varphi\left(q_x-\frac{\delta\lambda_\varphi}{\lambda}\right)+C_{90°}\left(\frac{\delta\lambda 90°}{\lambda}-r_x\right)\right\}\right]$$

We note in this equation that $\delta\lambda_\varphi > \delta\lambda 90^\circ$, $C_\varphi > C 90^\circ$, $q_x > r_x$; further as x increases from zero, $(q_x - r_x)$ diminishes and $(q_x \cdot x - r_x \cdot x)$ initially increases. Therefore R'_φ diminishes as x increases from zero and the curve R'_φ plotted against x begins to slope down towards the right. This was verified experimentally by the writer (Pal, 1949), for $\phi = 150^\circ$.

(b) In the forward direction, $\phi < 90^\circ$, the graph $R'_\varphi - x$ may be (i) ascending, (ii) descending, or (iii) horizontal, depending on the relative magnitude of the interference effect.

(i) *Ascending graph* : When the effect of interference is small—as for light scatterers and short wavelengths or large angle ϕ —as compared with the Compton short effect, we may put $f' \approx 0$, $\Delta_\varphi \simeq 0$ and $p_x \simeq 0$ in equation (24) and obtain

$$R'_\varphi = f(\phi, \lambda)\Big[\, 1 + B\frac{3}{2} \cdot x\lambda^3 \Big\{ C 90^\circ \Big(r_x + \frac{\delta\lambda 90^\circ}{\lambda} \Big) - C_\varphi \Big(q_x + \frac{\delta\lambda_\varphi}{\lambda} \Big) \Big\}$$

$$+ 3 \Big\{ C 90^\circ \Big(\frac{\delta\lambda 90^\circ}{\lambda} - r_x \Big) - C_\varphi \Big(\frac{\delta\lambda_\varphi}{\lambda} - q_x \Big) \Big\} \Big]$$

Here $\delta\lambda 90^\circ > \delta\lambda_\varphi$, $C 90^\circ > C_\varphi$ and $r_x > q_x$. Further, as x increases from zero, $(r_x - q_x)$ diminishes and $(r_x \cdot x - q_x \cdot x)$ initially increases. Therefore R'_φ initially increases with x, so that the curve $R'_\varphi - x$ is ascending towards the right.

(ii) *Descending graph* : When, however, the effect of interference is large—as for scatterers of high atomic number, long wavelengths or small angle ϕ—as compared with the Compton effect, the latter may be neglected and equation (24) may be written down as :

$$R'_\varphi = f(\phi, \lambda) \Big[\, 1 - \frac{3B}{2}\, x\lambda^3 \Big\{ \frac{\Delta_\varphi}{\lambda} + p_x \Big\} + 3p_x \,\Big]$$

Here as x increases from zero, p_x diminishes and $(p_x \cdot x)$ initially increases. Therefore R'_φ initially diminishes as x increases, so that the curve $R'_\varphi - x$ is descending towards the right.

(iii) *Horizontal graph* : The condition for horizontality requires that R'_φ should be equal to R_φ for all values of x concerned. Hence equating equations (24) and (25) it may be deduced that :

$$0 = -\frac{B}{2}\, x\,\lambda^3 \Big\{ \frac{\Delta_\varphi}{\lambda} + p_x + C_\varphi \Big(q_x - p_x + \frac{\delta\lambda_\varphi}{\lambda} \Big) - C 90^\circ \Big(r_x + \frac{\delta\lambda 90^\circ}{\lambda} \Big) \Big\}$$

$$+ \Big\{ -\frac{\Delta_\varphi}{\lambda} + p_x + C_\varphi \Big(q_x - p_x - \frac{\delta\lambda_\varphi}{\lambda} \Big) + C 90^\circ \Big(\frac{\delta\lambda 90^\circ}{\lambda} - r_x \Big) \Big\},$$

for all values of x.

Collecting the terms free from and independent of 'x' and equating them to zero, we have

$$\frac{\Delta_\varphi}{\lambda}+C_\varphi\frac{\delta\lambda_\varphi}{\lambda}-C_{90^\circ}\frac{\delta\lambda_{90^\circ}}{\lambda}=0 \qquad \dots \quad (26a)$$

Substituting this in the rest of the above expression which must also be equal to zero, we have

$$\left(1-\frac{B}{2}x\lambda^3\right)\{p_x+C_\varphi(q_x-p_x)-C_{90^\circ}r_x\}=0$$

for all values of x.

$$\therefore \qquad p_x+C_\varphi(q_x-p_x)-C_{90^\circ}r_x=0 \qquad \dots \quad (26b)$$

for all values of x concerned.

It is noted that the condition (26b) is identical with the condition (26a) when $x=0$.

Thus if the graph $R'_\varphi-x$ is to be horizontal, the general condition which must be satisfied is given by eqn. (26b). This relation must hold for values of x concerned from 0 upwards.

Cor. 1. If p_x is large, the above condition reduces to

$$p_x \simeq C_{90^\circ}r_x$$

Cor. 2. If $p_x=0$ (as in the backward direction) the condition (26b) can never be satisfied and no horizontal graph can be obtained.

Horizontality implies a constancy of the ratio R'_φ i.e. $(S'/P')_\varphi=$ Const. $\times(S'/P')_{90^\circ}$. Therefore, by suitably choosing the scale of reference for one set of ratios, the graph $(S'/P')\phi-x$ may be made to superpose exactly on the graph $(S'/P')_{90^\circ}-x$. This is illustrated for $\phi=30^\circ$ and 60° in Fig. 4. p. 652. Barkla and Khastgir (1926b).

All the above three varieties of the graph were obtained by the writer (Pal, 1949), for $\phi=30^\circ$ or 20°. Graph of the type (i) was yielded by carbon and paraffin was scatterers, of the type (iii) by filter paper and of the type (ii) by aluminium and sulphur, in complete accord with the theoretical results. What Barkla and Khastgir had recorded in Fig. 4, was but one variety which was equivalent to the third. The horizontal and the descending graphs had also been observed by Barkla and Mackenzie (1925—1926).

The results of the types (ii) and (iii) appear to be in conflict with the Compton effect. But this appearance, clearly enough, is only superficial. For the nature of the overall result is determined essentially not by the Compton effect alone, but by the combination and relative magnitudes of the Compton effect and interference. They operate upon the radiations concerned in opposite manners;

e.g. the scattered rays at $\phi = 30°$ are rendered harder than those at $\phi = 90°$, due to the Compton effect, whereas the reverse is the case due to interference.

SECTION III
COMPARISON OF THE SCATTERED AND THE PRIMARY RADIATIONS FOR DIFFERENT INCIDENT WAVELENGTHS

A. *Evaluation of the percentage difference between the intercepted and unintercepted ratios*

Let the ratio of the ionizations produced by the scattered and the primary radiations—both unintercepted—be denoted by $(S/P)_\varphi$ and the same when both the beams are intercepted by a constant thickness x of an absorber, by $(S'/P')_\varphi$.

Now, $(S/P)_\varphi$ can be calculated from eqn. (21) by putting $x = 0$, so that

$$(S/P)_\varphi = K'f(\phi, \lambda)\Big[1+C_\varphi \frac{f'}{f}\ \delta\lambda_\varphi+3\left\{p_0+C_\varphi\left(q_0-p_0-\frac{\delta\lambda_\varphi}{\lambda}\right)\right\}\Big]$$

$$= K'f(\phi, \lambda)\left[1+C_\varphi \frac{f'}{f}\ \delta\lambda_\varphi+3p_0\right],$$

since the quantity inside the small bracket contained within the curled bracket is equal to zero. With the help of this equation and equation (21), the fractional difference between the ratios i.e.

$$\{(S/P)_\varphi-(S'/P')_\varphi\}/(S/P)_\varphi$$

$$= \Big[3p_0+\frac{3B}{2}\, x\lambda^3\left\{\frac{\Delta_\varphi}{\lambda}+p_x+C_\varphi\left(q_x-p_x+\frac{\delta\lambda_\varphi}{\lambda}\right)-(A-A')x\right.$$

$$\left.-3\left\{p_x+C_\varphi\left(q_x-p_x-\frac{\delta\lambda_\varphi}{\lambda}\right)\right\}\right\}\Big]\Big/\left(1+3p_0+C_\varphi\frac{f'}{f}\,\delta\lambda_\varphi\right).$$

(a) If $x \geqq x_c$ *, then $p_x = 0$ and $q_x = 0$, so that

$$\{(S/P)_\varphi-(S'/P')_\varphi\}/(S/P)_\varphi = \Big[3p_0+\frac{3}{2}\, Bx\lambda^3\left\{\frac{\Delta_\varphi}{\lambda}+C_\varphi\ \frac{\delta\lambda_\varphi}{\lambda}\right\}$$

$$-(A-A')x+3C_\varphi\,\frac{\delta\lambda_\varphi}{\lambda}\Big]\Big/\left(1+3p_0+C_\varphi\frac{f'}{f}\,\delta\lambda_\varphi\right) \qquad \text{... (27)}$$

At $\phi = 30°$, $3p_0$ is large compared to all other terms except 1. Hence approximately

$$\{(S/P)_\varphi-(S'/P')_\varphi\}/(S/P)_\varphi = 3p_0/(1+3p_0)$$

$$= \frac{3\,\Delta_\varphi}{\lambda}\Big/\left(1+\frac{3\Delta_\varphi}{\lambda}\right) \qquad \text{... (28)}$$

* x_c corresponds to q_x ($>$ p_x)

Δ_φ increases as λ increases. Now if we assume that $\Delta_\varphi \propto \lambda$, to a rough approximation, for a small range of wavelengths at least, then it is at once evident that the above fractional difference and hence the percentage difference between the intercepted and unintercepted ratios is constant, i.e. independent of the incident wavelength λ within that range.

The percentage difference, when computed from the above, may also come out with the right order of magnitude, as the following tentative example will show.

Example : Let $\lambda = 0.6\,\text{Å}$ (which is approximately equal to the mean of the wavelengths employed by Barkla and Khastgir) and $\Delta_\varphi = 0.020\,\text{Å}$ on a modest estimate. Then the percentage difference between the ratios is given by :

$$100 \cdot \frac{(S/P)_\varphi - (S'/P')_\varphi}{(S/P)_\varphi} = 300 \times \frac{0.020}{0.6}\left(1 - 3 \times \frac{0.020}{0.6}\right) = 9\%.$$

This agrees exactly with that estimated by Barkla and Khastgir (1962b) (*Vide* Fig. 5. p. 653) showing that the agreement is not only qualitative but may be quantitative as well.

(b) Constancy of the percentage difference between the ratios for $\phi = 90°$ and $60°$.

We have already learnt that when the incident radiation satisfies the conditions expressed in (26 a) and (26b), R'_φ is constant and independent of x.

$\therefore$ Taking the log. differential with respect to x, $\delta\{\log R'_\varphi\} = 0$.

or $\quad \delta[\log\{(S'/P')_\varphi/(S'/P')_{90°}\}] = 0.$

or $\quad \delta[\log\{(S'/P')_\varphi - \log(S'/P')_{90°}\}] = 0.$

or $\quad \delta(S'/P')_\varphi/(S'P')_\varphi = \delta(S'/P')_{90°}/(S'/P')_{90°}$

Now, considering the variation of the ratios between $x = 0$ and $x = x$ (small), we may rewrite the above in the form

$$100[(S/P)_\varphi - (S'/P')_\varphi]/(S/P)_\varphi = 100[(S/P)_{90°} - (S'/P')_{90°}]/(S/P)_{90°}$$

Putting $\phi = 60°$ and $30°$ successively, we have

$$\frac{100\{(S/P)_{60°} - (S'/P')_{60°}\}}{(S/P)_{60°}} = \frac{100\{(S/P)_{90°} - (S'/P')_{90°}\}}{(S/P)_{90°}}$$

$$\frac{100\{(S/P)_{30°} - (S'P')_{30°}\}}{(S'/P)_{30°}} \qquad (29)$$

Thus the percentage difference between the intercepted and unintercepted ratios for $\phi = 90°$ and the same for $\phi = 60°$, are equal and each equal to the corresponding percentage difference for $\phi = 30°$ (which has been shown to be a constant-independent of λ through a small range of wavelengths, when $x \lesseqgtr x_c$). This quality, of course, is valid so long as the particular wavelength employed satisfies the condition (26a) i.e.

$$\Delta_\varphi + C_\varphi \delta\lambda_\varphi = C_{90^\circ}\ \delta\lambda_{90^\circ}, \text{ for } \phi = 30^\circ \text{ and } 60^\circ.$$

The other condition (26.b) is automatically satisfied on account of $x \gtreqless x_0$.

The validity mentioned above should persist at least over a small range of wavelengths. For, though with a change in λ the equality expressed in (26a) is apparently disturbed (since with an icrease in λ. Δ_φ increases while C_φ and C_{90° decrease), yet any small unbalance should be counteracted and set right by a corresponding change in absorption occuring inside the scatterer, over that small range of wavelength—of the order of 0.1 Å (as in Barkla and Khastgir's experiments —*vide* Fig. 5, Barkla and Khastgir (1926b). Over this range, therefore, the percentage difference between the ratios for $\phi = 60^\circ$ and 90° will be each equal to that for $\phi = 30^\circ$, which is again maintained constant ($= 9\%$) through that range. This result is also in accord with Barkla's experimental observation.

But there is yet another point to consider. Beyond this range in question and on the longer wavelength-side, the equation (26a) which was previously fulfilled may break down owing to C_{90° and C_φ having rapidly fallen to a zero value at a particular wavelength and the scattered radiation, as a whole, becoming unmodified thereby. In that case our above treatment and deduction fall through. To calculate the corresponding percentage difference of the ratios for $\phi = 90^\circ$, it is necessary to go back to equation (27), where we have to put $p_0 = \Delta_\varphi/\lambda = 0$, $C_\varphi = 0$ and $(A-A')\cdot x \gtrsim 0$. This brings down the required percentage difference to a zero value, which explains the zero-line graph in Fig. 5, case $A.$, p. 653, Barkla and Khastgir (1926b).

An analogous reasoning applies to $\phi = 60^\circ$. Putting $C_{60^\circ} = 0$ and remembering that the interference effect for paraffin wax scatterer at this large angle, is small indeed, we may assess from eqn. (27) a magnitude for the percentage difference between the ratios at hardly above 1 or 2% which falls close to the limits of experimental error, thus yielding the zero-line graph again, as in the case of $\phi = 90^\circ$ above.

A similar argument when λ is large, however, is not tenable in the case $\phi = 30^\circ$; for, the calculation in (28) was independent of the condition (26a). The zero-line graph appearing above, is therefore, missing in this case.

The fact that the percentage difference between the ratios in (29) is equal to either a constant or zero, is equivalent to the law of the J-phenomenon :

$$\left(\frac{\mu}{\rho}\right)_s - \left(\frac{\mu}{\rho}\right)_p = a \text{ (const.) or } 0.$$

for the direction concerned. [See also equations (30)* and (30(*a*))]

Thus may be explained, in a general way, the features of the graphs presented in Fig. 5., p. 653, Barkla and Khastgir (1926b), viz.

(i) The double horizontal lines showing modified and unmodified scattering, separately for $\phi = 90°$ and $60°$ and a single horizontal line standing for both modified and unmodified scattering, for $\phi = 30°$.

(ii) The degree of modification is independent of the angle of scattering ϕ and of the original incident wavelength λ (within a small range) under suitable circumstances.

(iii) The discontinuity (J_1) setting in at about the same wavelength (~ 0.6 Å) for $\phi = 90°$ and $\phi = 60°$.

B. *Scattering at $\phi = 90°$ and the horizontal graph ($S/P-\lambda$) for the unintercepted ratio*

The intercepted ratio obtained in eqn.(29) reads as :

$$(S'/P')_{90°} = K'\left[1-3C_{90°}\left\{\frac{B}{2}\, x\,\lambda^3\left(r_x+\frac{\delta\lambda_{90°}}{\lambda}\right)+\left(\frac{\delta\lambda_{90°}}{\lambda}-r_x\right)\right\}+(A-A')x\right]$$

Putting $x = 0$ the unintercepted ratio is deduced as :

$(S/P)_{90°} = K'$ * (which is a constant independent of λ) (30)

since $\frac{\delta\lambda_{90°}}{\lambda}$

Therefore $(S/P)_{90}$ plotted against λ or $(\mu/\rho)_{AL}$ gives a horizontal line graph agreeing with Barkla and Khastgir's observation (1927).—*vide* Figs. 1 and 2 p. 739.

C. *Absorption* : $\phi = 90°$.

(a) Absorbing thickness constant and greater than critical: When $x > x_c$ (where x_c corresponds to the smallest wavelength employed) and in addition the condition (26a) is fulfilled over a small range of wavelengths, we know from (29) that over this range

$$100\cdot\{(S/P)_{90°}-(S'/P')_{90°}\}/(S/P)_{90°} = G, \text{ say, (const.)}.$$

* An identical result may be obtained in the case of a monochromatic beam also. For, we can pass over from the heterogeneous beam to the homogeneous, by putting $r_x = \frac{\delta\lambda_{90°}}{\lambda}$

This again makes $(S/P)_{90°} = K' =$ Const.

Regarding the heterogeneous incident spectrum as made up of a multitude of monochromatic constituents, represented by numerical subscripts, we have at $\phi = 90°$,

$$\begin{aligned} K' &= (S_1/P_1) = (S_2/P_2) = (S_3/P_3) \ldots. \\ &= (S_1+S_2+S_3+\ldots.)/(P_1+P_2+P_3+\ldots.) \\ &= (S/P)_{90°} \text{ which is the same as (30),} \end{aligned}$$

where P_n and S_n denote actual amounts of ionization produced by primary and scattered beams respectively corresponding to the n-th constituent.

Substituting the value of $(S/P)_{90°} = K'$ from (30), we have

$$(S'/P')_{90°} = K' \cdot [1 - G/100] = 0.91K', \qquad \ldots \quad [30(a)]$$

assuming $G = 9\%$ as obtained by Barkla and Khastgir.

Thus the intercepted ratio is also constant over the above range of wavelengths, giving another horizontal line below the first, corresponding to the unintercepted ratio. Here also, the percentage difference between the two ratios should be the same (9% as previously) and was actually found to be so, irrespective of the material* of the absorber, in accordance with theory.

There is a marked bending down of the two graphs mentioned above towards the left-hand side, where presumably, the incident radiations were those which were hardened more and more by progressive filtration of a certain beam. In this region, the relation (26a) ceases to be valid and the ratio $(S'/P')_{90°}$ has to be recalculated from equation (22). Remembering that $r_x = 0$ and $(A - A')\cdot x \simeq 0$, we have

$$(S'/P')_{90°} = K'\left[1 - 3C_{90°}\,\frac{\delta\lambda_{90°}}{\lambda}\left(1 + \frac{B}{2}\,x\lambda^3\right)\right]$$

$$\simeq K'\left[1 - 3C_{90°}\,\frac{\delta\lambda_{90°}}{\lambda}\right], \text{ since } \frac{B}{2}\,x\lambda^3 << 1.$$

Now, as λ decreases $C_{90°}$ increases. Therefore $(S'/P')_{90°}$ decreases as λ decreases, which accounts for the observed bending of the graph for the intercepted ratio.

(b) Absorbing thickness constant, but very small: When x is very $(x < x_s)$, $r_x \simeq \dfrac{\delta\lambda_{90°}}{\lambda}$ **, so that equation (22) becomes :

$$(S'/P')_{90°} = K'\left[1 - 3C_{90°}\left\{Bx\lambda^3\,\frac{\delta\lambda_{90°}}{\lambda}\right\} + (A - A')x\right]$$

$$= K'\left[1 - x\left\{3C_{90°}\,B\lambda^3\,\frac{\delta\lambda_{90°}}{\lambda} - (A - A')\right\}\right]$$

Referring to Fig. 5., p. 744, Barkla and Khastgir (1927) and considering the case of Ag-absorber of thickness $x = 0.006$ cm., we may form from the above, an estimate of the ratio $(S'/P')_{90°}$ for the same.

* A small difference—about 1%—consistently shown by the Cu and Al lines (intercepted) in fig. 1, p. 739, Barka and Khastgir (1927) may be due to the fact that $(A - A')x$ is equal to 0 for Cu and about 0.15 for Al (neglected previously).

** For long wavelengths.

Putting $x = 0.006$ cm., $B = 84.6$, $\delta\lambda_{90°} = 0.024$ Å, $\lambda = 0.5$ Å (mean value) and tentatively, $C_{90°} = 0.5$ and also $(A - A') = 0.3$, one obtains

$$x \left\{ 3C_{90°} B\lambda^3 \frac{\delta\lambda_{90°}}{\lambda} - (A - A') \right\} = 0.003 \text{ (approx)}$$

i.e. $$\left(\frac{S'}{P'}\right)_{90°} = K' \quad \text{(correct to about } 0.3\%\text{)}$$

$$= \left(\frac{S}{P}\right)_{90°}$$

Thus the intercepted graph $(S'/P')_{90°} - \lambda$ for $Ag (x = 0.006$ cm) superposes itself on the unintercepted horizontal graph $(S/P)_{90°} - \lambda$, agreeing with the experimental observation of Barkla and Khastgir. The appearance of the unmodified scattering here (see eqn. 31), is simply superficial and illusory for obvious reasons.

The companion parallel straight line for Al in the same figure being 9% below the unintercepted ratio—line. points to the conclusion that its thickness (0.15 cm) is either equal to or greater than the critical corresponding to the smallest wavelength concerned and that the condition (26a) has been realised over the experimental range of wavelengths. The thickness of Ag (0.006 cm)is certainly less than its critical value; for otherwise, would its graph superpose on that of Al.

For hard filtered radiations, however, the picture appears to be entirely different. In that case, r_x is no longer approximately equal to $\frac{\delta\lambda_{90°}}{\lambda}$ and $\left(\frac{\delta\lambda_{90°}}{\lambda} - r_x\right)$ is not negligible as in (b).

The equation (22), therefore, reduces to

$$(S'/P')_{90°} = K' \left[1 - 3C_{90°} \left(\frac{\delta\lambda_{90°}}{\lambda} - r_x \right) \right],$$

since the missing terms all work up together to a negligible magnitude. As the wavelength λ is decreased, $C_{90°}$ increases (up to a value 1) and so also $\left(\frac{\delta\lambda_{90°}}{\lambda} - r_x\right)$ *vide* (*f*) of "Properties of q_x", with the result that the intercepted

This is the interpretation of the marked decline in the ratio $(S/P)_{90°}$ towards the ratio falls. harder end of the graph [in figs. 1 and 2, p. 739 (Barkla and Khastgir 1927)]. Because, though it is unintercepted in the ordinary sense, the rays concerned have yet to pass through a very small thickness (0.01 cm) of Al which covers the window of the ionization chambers.

(c) Absorption of unmodified scattered radiation : At large values of λ, the scattered radiation is unmodified and $C_{90°} = 0$. Hence from (22)

$$(S'/P')_{90°} = K'[1 + (A - A')x] = K'(\text{const.})$$

$$= (S/P)_{90°}, \text{ since } (A - A')x << 1. \quad \dots \quad (31)$$

This relation thus becomes independent of the material of the absorber to some extent and also of its thickness.

Therefore, the graph for the intercepted ratio plotted against λ lies on the same horizontal straight line as the corresponding graph for the unintercepted ratio, which is in conformity with the classical theory of scattering. The feature in question, is clearly borne out by the plots in the longer wavelength region $-(\mu/\rho)_{Al} > 3.6$ in Fig. 5., p. 653, Barkla and Khastgir (1926b) and in Figs. 3 and 4, pp. 1121 and 1123, Barkla and Khastgir (1925b).

(d) Non-commutative absorption.

Referring to eqn. (23) in connection with the compound absorbers we have

$$(S''/P'')_{x+y} = K' \Big[1 - 3C_{90^\circ} \Big\{ \frac{B_1}{2} x\lambda^3 \Big(r_x + \frac{\delta\lambda_{90^\circ}}{\lambda} \Big) + \Big(\frac{\delta\lambda_{90^\circ}}{\lambda} - r_x \Big) \Big\} + (A_1 - A_y')x$$

$$-3C_{90^\circ}\Big\{ \frac{B_2}{2} y\lambda_x^3 \Big(\bar{r}_y + \frac{(\delta\lambda_{90^\circ})_x}{\lambda_x} \Big) + \Big(\frac{(\delta\lambda_{90^\circ})_x}{\lambda_x} - \bar{r}_y \Big) \Big\} + (A_2 - A'_2)y \Big]$$

Interchanging the positions of the two absorbers, we have .

$$(S''/P'')_{y+x} = K' \Big[1 - 3C_{90^\circ} \Big\{ \frac{B_2}{2} y\lambda^3 \Big(r_y + \frac{\delta\lambda_{90^\circ}}{\lambda} \Big) + \Big(\frac{\delta\lambda_{90^\circ}}{\lambda} - r_y \Big) \Big\} + (A_2 - A'_2)y$$

$$-3C_{90^\circ}\Big\{ \frac{B_1}{2} x\lambda_y^3 \Big(\bar{r}_x + \frac{(\delta\lambda_{90^\circ})_y}{\lambda_y} \Big) + \Big(\frac{(\delta\lambda_{90^\circ})_y}{\lambda_y} - \bar{r}_x \Big) \Big\} + (A_1 - A'_1)x \Big]$$

Obviously, the ratio $(S''/P'')_{x+y}$ is not identical with the ratio $(S''/P'')_{y+x}$, except when the rays are homogeneous or when the absrobers are of the same material. This is an interesting result, pointing to the importance of the order or sequence in which the absorbers are placed.

It must be noted here, that although the experimental findings of Barkla and Khastgir have, in this paper, been amply corroborated from the theoretical stand-point, yet these findings are not to be accepted as perfectly general. The features in question owe their origin to a particular set of conditions imposed upon the experiments, which under other conditions, may lead to a different picture altogether and that with equal theoretical justification. Indeed, performances of heterogeneous X-rays, have shown themselves to be extremely sensitive to these conditions and variation thereof, brings about appreciable changes in the consequences. The choice of the absorbing thickness, for instance, is often a vital factor in deciding the issue. Many more examples, to this effect, may be cited—some from the writer's own works (unpublished) also. In one experiment, as previously mentioned, progressive absorption by *Ag* (maximum thickness = .0095 cm) yielded a curve smoothly sloping down, for $\lambda = 0.46$ Å, in striking contrast with the horizontal line obtained by Barkla and Khastgir. In another experiment, the ratio $(S/P)_{90}$ plotted against $(\mu/\rho)_{Al}$ traced out a graph which ran horizontal for a certain length but steadily sloped down towards the softer region of the rays. This was accompanied by the corresponding $(S'/P')_{90}$ —graph for 0.7 mm Al-absorber, which was definitely a descending curve and clearly distinct from the horizontal type, discussed previously. Eve the thickness of the scatterer has been noticed to play its part and affects th ensuing results substantially. Yet none of these deviations have so far run co nter with the concepts presented in the foregoing pages. Notwithstanding the act that some of the experimental results were at variance (due undoubte to changed conditions) with those under review, there were still, a host of oth rs in line with them.

It is, however, realised that the type of absorption experiments dealt with in this paper, despite possessing certain advantages from experimental point of view, is hardly the type best suited for a crucial test of the Compton effect or of the concept of independent quanta. The reason is that, the individual constituents of the incident spectrum do not act incoherently. Another thing that should be borne in mind in this context is that, the very structure of the heterogeneous complex radiation continually undergoes metamorphosis by absorption, as it passes from layer to layer of the absorbing substance, which considerably complicates the issue. Nevertheless, the present analysis, founded essentially on the validity of the Compton's quantum theory of X-ray scattering, has been able to explain most of the results experimentally observed and to substantiate many of the features of the J-phenomenon.

SUMMARY AND DISCUSSION

Starting with Barkla's idea of an 'atmosphere' of radiation, mentioned before and taking into account the various effects due to (i) Compton scattering, (ii) interference of the scattered radiations, (iii) absorption of the various constituents of the heterogeneous beam by an absorbing material and (iv) ionization produced by the radiations received in the ionization chambers, it has been possible to explain qualitatively and also quantitatively in many cases, most of the experimental results of Barkla and Khastgir on the comparative study of the primary and the scattered beams of heterogeneous X-radiations, (i) when both the beams were passed through the same increasing thickness of an absorbing substance for a particular average absorbability of the primary beam and (ii) when both the beams were intercepted by the same thickness of an absorbing substance for a wide range of absorbabilities of the primary beam.

With regard to the Compton scattering, the modified and the unmodified parts have been considered separately. In considering the effects of interference of the scattered radiations, predominantly in the forward direction, enhancement of the scattered intensity and the increase in the wavelength of the scattered beam have been taken into account. A function f depending on the wavelength

5

and the angle of scattering has been introduced to represent the enhanced intensity due to interference and the increase in the wavelength due to interference has been considered in addition to the Compton change of wavelength.

In the experiments with which we are concerned, the scattered beam was fairly wide in comparison with the primary beam, so that the primary and the scattered beams could be comparable in their ionizing effects. With a narrow beam, the mass -absorption coefficient is slightly higher than that with a wide beam. This increased mass-absorption coefficient of the narrow primary beam relative to the wide scattered beam has been taken into account in the theoretical analysis. In calculating the ratio S'/P' for the heterogeneous beam, the concept of a critical absorbing thickness has been very useful. Further, a 'disparity" —term, q_x, has been introduced to characterise the difference in quality between two beams of slightly different wavelengths after transmission through a certain thickness x of an absorbing material. The average wavelength inside the absorbing material has been taken as the mean of the average incident wavelength and the average transmitted wavelength, and the 'disparity' between the two beams of slightly different wavelengths after transmission has been defined as the fractional difference in the average transmitted wavelengths. The 'disparity'—term has simplified the problem of relative absorption by two beams of slightly different wavelengths, while dealing with heterogeneous X-rays. Finally the ionization produced by the radiations transmitted through an absorbing material and received in the ionization chambers has been considered.

(a) *Progressive Absorption*

Considering all the above factors, a theoretical expression for the ratio of the ionizations due to the scattered and the primary beams after transmission through a gradually increasing thickness of an absorbing material has been worked out for a definite average wavelength of the primary beam. This has enabled to explain the experimental results of Barkla and Khastgir on progressive absorption of the scattered and the primary beams of heterogeneous X-rays. The observations that in the case of Al and Cu, the ratio of the ionizations produced by the scattered and the primary beams decreased with the increasing thickness of the absorbing substance and that in the case of Ag and Sn, the ratio was independent of the thickness of the absorber have been interpreted from theoretical considerations.

The progressive absorption of the two beams of heterogeneous X-rays, scattered at any angle ϕ and scattered at 90° has also been theoretically investigated. It has been shown that in the backward direction ($\phi > 90°$), the ratio of the ionizations due to the two scattered beams should decrease with the increase of thickness of the absorbing substance and that in the forward direction ($\phi < 90°$), the ratio should (i) decrease as in the case of backward direction or (ii) increase or (iii) remain constant with the increasing thickness. All the three types of the variation were actually observed by the author (Pal, 1949). The results of Barkla and Mackenzie (1925, 1926) corresponded to the types (i) and (iii) and those of Barkala and Khastgir corresponded to the type (i) only.

When a decrease in the ratio of the ionizations due to the scattered and primary beams with increasing thickness of the absorbing substance was observed, Barkla and Khastgir occasionally observed a reduced rate of decrease. This 'sub-level' representing a smaller slope of the curve has been attributed by the author to the instability of the scattered beam in course of the observations. An attempt has also been made to explain the apparently anomalous results which had been obtained by Barkla and Khastgir (1926b) with compound absorbers (i.e. with a gradually increasing thickness of Al backed by a constant thickness of Ag or Sn).

(b) *Ratio of the ionizations due to the scattered and the primary beams, intercepted and unintercepted by a given thickness of the absorbing substance for different mass-absorption coefficients of the incident primary beam.*

The precentage difference between the intercepted and the unintercepted ratios has been theoretically worked out. Assuming that the change in wavelength Δ_{φ} due to interference of the scattered radiations is directly proportional to the wavelength λ (which is true to a first approximation) for a small range of wavelengths, it has been shown that the percentage difference, as computed theoretically is constant, i.e. independent of the incident wavelength λ within a certain range, provided the thickness is equal to or greater than what has been called the critical absorbing thickness. This is what had been observed by Barkla and Khastgir after the J_1-discontinuety had set in as in their Case A or throughout the range as in their Case B. It has also been shown that the computed percentage difference between the 'unintercepted' and the 'intercepted' ratios is of the same order of magnitude as had been observed by Barkla and Khastgir (i.e. 9%, when Al was used to intercept the beams). Further, the significant observation of Barkla and Khastgir that the percentage difference between the 'unintercepted' and the 'intercepted' ratios is of the same order for the angles of scattering 30°, 60° and 90°, has been explained, under certain conditions, from theoretical considerations.

The constancy of the percentage difference between the two ratios S/P and S'/P', over a range of wavelengths implies that difference between the mass-absorption-coefficients of the primary and the scattered beams is also constant over the same range of wavelengths. This difference is of course zero in Case A of Barkla and Khastgir for longer wavelengths. We thus get some theoretical justification for Barkla's law of the J-phenomenon :

$$\left(\frac{\mu}{\rho}\right)_1 - \left(\frac{\mu}{\rho}\right)_2 = 0 \text{ or constant.}$$

The theoretical analysis has also shown that the 'unintercepted' and the 'intercepted' ratios of the scattered and the primary beams, under certain conditions are both constant and independent of wavelength. The well-known horizontal lines of Barkla and Khastgir in their curves showing S/P and S'/P' for different mass-absorption coefficients of the primary beam are thus explained. A marked bending down of the lines for very small mass-absorption coefficients has also been interpreted. In the case of compound absorbers, a striking result of the theoretical analysis is the inequality of $((S''/P'')_{x+y}$ and $(S''/P'')_{y+x}$ for a heterogeneous beam of X-rays, where x and y are the thicknesses of two different absorbing substances, in contact with each other, once when placed one behind the other and subsequently, when placed in the reversed order. This non-commutative absroption needs an experimental verification.

ACKNOWLEDGMENT

The writer expresses his indebtedness to late Prof. C. G. Barkla, D.Sc., F.R.S., N.L. for affording him all facilities for experimental work in his laboratory at Edinburgh, U.K. and for giving him helpful guidance during 1935-37. His best thanks are also due to Prof. S. R. Khastgir, D.Sc., F.N.I., Head of the Dept. of Physics, Bose Institute, Calcutta, for valuable discussions and assistance in writing the paper.

REFERENCES

Bachem, A., 1923, Principles of X-ray and Radium Dosage, Chicago, p. 152*et seq.*
Backhurst, I., 1934, *Phil. Mag.*, Vol. XVII, 321.
Barkla, C. G., 1924, *Nature*, Vol. CXIV, 753.
Barkla, C. G., 1925, *Phil. Mag.*, Vol. XLIX, 1033.
Barkla, C. G., 1926, *Nature*, Vol. CXVII, 448.
Barkla, C. G., 1927, *Nature*, Vol. CXIX, 778.
Barkla, C. G., 1928, *Phil. Mag.*, Vol. V, 1164.
Barkla, C. G., 1929, *International Critical Tables*, Vol. VI, 1.
Barkla, C. G., 1931, *Nature*, Vol. CXXVII, 877.
Barkla, C. G., 1933, *Nature*, Vol. CXXXI, 166.
Barkla, C. G. and Khastgir, S. R., 1925a, *Phil. Mag.*, Vol. XLIX, 251.
Barkla, C. G. and Khastgir, S. R., 1925b, *Phil. Mag.*, Vol. I, 1115.
Barkla, C. G. and Khastgir, S. R., 1926a, *Nature*, Vol. XCVII, 228.
Barkla, C. G. and Khastgir, S. R., 1926b, *Phil. Mag.*, Vol. II, 642.
Barkla, C. G. and Khastgir, S. R., 1927, *Phil. Mag.*, Vol. IV, 735.
Barkla, C. G. and Mackenzie, G., 1925, *Nature*, Vol. CXV, 942.
Barkla, C. G. and Mackenzie, G., 1926, *Phil. Mag.*, Vol. I, 542.
Barkla, C. G. and Mackenzie, G., 1926, *Phil. Mag.*, Vol. II, 1116.
Compton, A. H., 1924, *Nature*, Vol. CXIII, 160.
Dirac, P. A. M., 1926, *Proc. Roy. Soc.*, *A*, Vol. III, 406.
Khastgir, S. R., 1932, *Phil. Mag.*, Vol. XIV, 99.
Pal, H. K., 1948, *Ind. Jour. Phys.*, Vol. **22**, Part VII, July , 291.
Pal, H. K., 1949, *Ind. Jour. Phys.*, Vol. **23**, No. 3, March, 111.
Pal, H. K., 1950, *Ind. Jour. Phys.*, Vol. **24**, No. 2, Feb., 91.
Pal, H. K., 1964, *Ind. Jour. Phys.*, Vol. **38**, No. 2, Feb., 61.